MATH 1300, Mathematical Explorations

Pennies and Paperclips

"In mathematics, there is the concept of proving something; of knowing it with absolute certainty, which is called rigorous proof. Rigorous proof is a series of arguments based on logical deductions which build one upon the other, step-by-step until you get to a complete proof. That's what mathematics is about." —Simon Singh

Three or Four 75-minute classes

Activity

- Distribute blank paper and explain set up. Then give students time to play several games.
- Ask them if they notice any patterns/strategies that help one player win? Then ask them to play some more games to test their strategies.
- Have them make a conjecture about when pennies will win based on how the pennies are placed on the board. Can they prove their conjecture?
- Play game on a new board: Rectangle with four extra squares at the end of the top row.
- Next play game on a 'frame:' A $2k \times 2k$ square with central $2(k-1) \times 2(k-1)$ square removed.
- Ask them to play the $2k \times n$ game again: put down the pennies on two different squares. Then take it in turns to put down the paper clips. One player tries to cover all the squares. The other tries to mess it up. Ask them to reflect on the 'proof'.
- Repeat this with the $2 \times n$ rectangle.
- Ask them to find Hamiltonian cycles in a $2k \times n$ checkerboard (with $n > 1$).
- Ask them to play the original pennies and paperclips game on the board with the Hamiltonian cycle drawn on it.
- Can they come up with a new 'proof' of the conjecture.

Questions for class

- What have you noticed?

- Is this math? Why or why not?
- What makes something a proof (versus an idea or hunch or evidence).
- Is the proof using hamiltonian cycles better than the first proof given for the pennies and paperclips game?

Notes

Students struggle to understand why their initial explanations of their conjectures are not proofs.

Once students play the game with Hamiltonian cycles on the board they construct a proof of their conjecture faster.

Assignments

Portfolio Assignement:

- Complete your portfolio entry about this week's classes while the material is still fresh in your mind.
- Today we discussed how to prove the following fact: On the $2k \times n$ grid, if the pennies are on two squares of opposite color, then paperclip wins. Discuss the proof as you understand it. Use the opportunity to reflect on what makes it a proof (versus an idea or conjecture or hunch).
- Besides what we discussed in class, here's another thing you could think about in your portfolio: is it possible to efficiently check all the possible configurations of pennies and show that paperclip always has a strategy?

References and resources

Pennies vs. Paperclips Worksheet

Why Do We Have to Learn Proofs!?

Follow-on activities

Find an infinite/bi-infinite Hamiltonian path in an infinite checkerboard.

Find Hamiltonian cycles in Platonic solids.